Solutions for Chapter 2

Exercise 2.1

a. Let h_0, h_1 be the switch logic for function x and g_0, g_1 the switch logic for function y. The index denotes, which logical value, 0 or 1, should be transferred. The Karnaugh–diagrams for both functions x and y describe the switch logic functions h_1 and g_1, respectively.

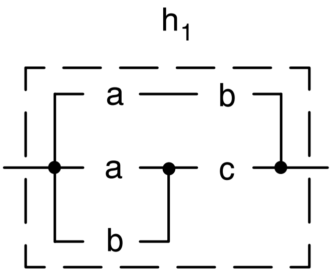
$x = h_1$		ab = 00	01	11	10
c	0	0	0	1	0
	1	0	1	1	1

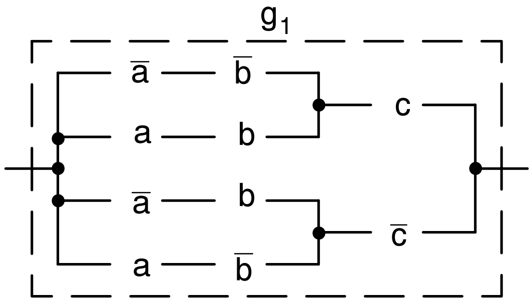
$y = g_1$		ab = 00	01	11	10
c	0	0	1	0	1
	1	1	0	1	0

$$h_1 = ab \vee ac \vee bc \qquad = ab \vee (a \vee b)c$$

$$g_1 = \bar{a}\bar{b}c \vee \bar{a}b\bar{c} \vee abc \vee a\bar{b}\bar{c} = (\bar{a}\bar{b} \vee ab)c \vee (\bar{a}b \vee a\bar{b})\bar{c}$$

The funktion h_1 comprises three blocks of two literals, whereas g_1 comprises four blocks of three literals. By factoring out the common variables c and $\bar{c}$ the following switch functions can be derived:





The switch functions h_0 and g_0, respectively, define the transfer of logical value 0, i.e., when the logical function equals 0 the switch function is 1.

$\bar{x} = h_0$		ab = 00	01	11	10
c	0	1	1	0	1
	1	1	0	0	0

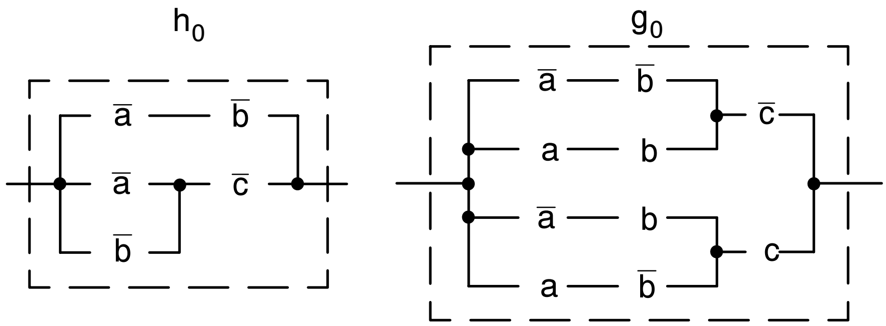
$\bar{y} = g_0$		ab = 00	01	11	10
c	0	1	0	1	0
	1	0	1	0	1

Using the minterms for h_0 and g_0, respectively, the following functions can be derived:

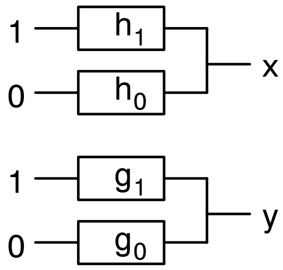
$$h_0 = \overline{a}\overline{b} \vee \overline{a}\,\overline{c} \vee \overline{b}\overline{c} \qquad = \overline{a}\overline{b} \vee (\overline{a} \vee \overline{b})\overline{c}$$

$$g_0 = \overline{a}\overline{b}\overline{c} \vee \overline{a}bc \vee ab\overline{c} \vee a\overline{b}c = (\overline{a}\overline{b} \vee ab)\overline{c} \vee (\overline{a}b \vee a\overline{b})c$$

leading to:



In this particular case the switch functions h_0 and g_0 have the same structure as h_1 and g_1.
The overall function is given by:



b. The Karnaugh−diagram taken from a. can be transformed in a value entered diagram using c and $\overline{c}$ as entered variables.

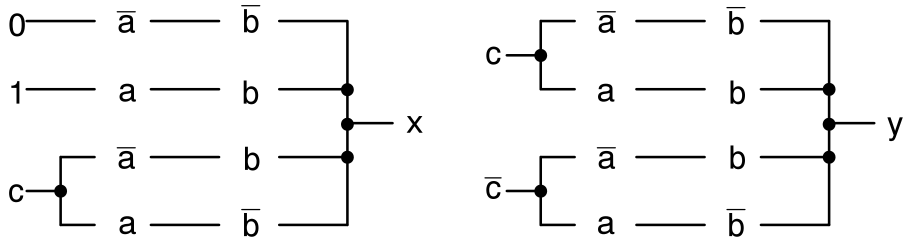
x \ ab	00	01	11	10
	0	c	1	c

y \ ab	00	01	11	10
	c	$\overline{c}$	c	$\overline{c}$

The functions above need switch logic functions h_0', h_1', h_c', $h_{\overline{c}}'$ and g_0', g_1', g_c', $g_{\overline{c}}'$, respectively.
Using the diagrams one obtains:

$$h_0' = \overline{a}\overline{b} \qquad g_c = \overline{a}\overline{b} \vee ab$$
$$h_1' = ab \qquad g_{\overline{c}} = \overline{a}b \vee a\overline{b}$$
$$h_c' = \overline{a}b \vee a\overline{b}$$

resulting in the switch logic functions:



c. The solution taken from part a. already represents a complex gate implementation. Only the switch functions for the transmission of value 1 and 0 have to be realized using p−channel and n−channel transistors, respectively. The p−channel transistors have to be controlled by complementary signals. Pseudo−NMOS can be implemented by h_0 and g_0, respectively, and an additional pull−up transistor.

d. Under the assumption that all the signals and their complements are available the transistor count is as follows:

n_{Tr}	x	y
Input signals 0,1	10	20
Input signals 0,1,c,$\bar{c}$	8	8
Complex gates	10	20
Pseudo−NMOS	6	11

Exercise 2.2

a.

a	b	y
0	0	z
0	1	0
1	0	z
1	1	1

z means: path is in tristate mode, the level of output voltage is determined by the charging of the output capacitor.

Erroneous behaviour for $b = 0$.

b.

a b	y
0 0	0
0 1	0
1 0	0
1 1	1

The logical function corresponds to the logical AND–function. The additional transistor guarantees a definite output level for b=0.

c. The two multiplexors mutually ensure a definite output level.

Exercise 2.3

a. T_1 saturation, because $u_{G1} = u_{D1} = U_{DD}$
T_2 linear operation, because $u_X = u_{D2} < U_{DD} - U_T$

b. $i_{DS1} = \frac{\beta}{2}(U_{DD} - u_X - U_T)^2$

$$i_{DS2} = \beta\left[(U_{DD} - u_{out} - U_T)(u_X - u_{out}) - \frac{(u_X - u_{out})^2}{2}\right]$$

Solution: $i_{DS1} = i_{DS2}$ for $u_{out} = 0$

$u_X = (U_{DD} - U_T)(1 - 1/\sqrt{2})$
Condition for transistor switch off:

$$i_{DS1} = 0 \Rightarrow U_{DD} - u_X - U_T = 0$$
$$u_X = U_{DD} - U_T$$
$$i_{DS2} = 0 \Rightarrow \quad u_X - u_{out} = 0$$
$$u_{out} = u_X$$

and finally $u_{out} = U_{DD} - U_T$

c. Using the solution from part b. we get:

$u_{G3} = U_{DD} - U_T$

$u_{out} = u_{S3} = U_D - 2U_T$

Conclusion: The output of a chain of pass–transistors should not be used to control other pass–transistors, because of the reduction of the voltage output level.

Exercise 2.4

a. $u_{DSn} = u_C\,;\; u_{GSn} = u_{in}$

Saturation: $u_C > (u_{in} - U_{Tn})/2$

$u_{in} < 2u_C + U_{Tn}$

$i_{DS,Sat} = \beta/2\,(u_{in} - U_{Tn})^2$

b. Linear operation:

$n:\quad U_{DSn} \leq (U_{GSn} - U_{Tn})/2$

$p:\quad -U_{DSp} \leq (-U_{GSp} + U_{Tp})/2$

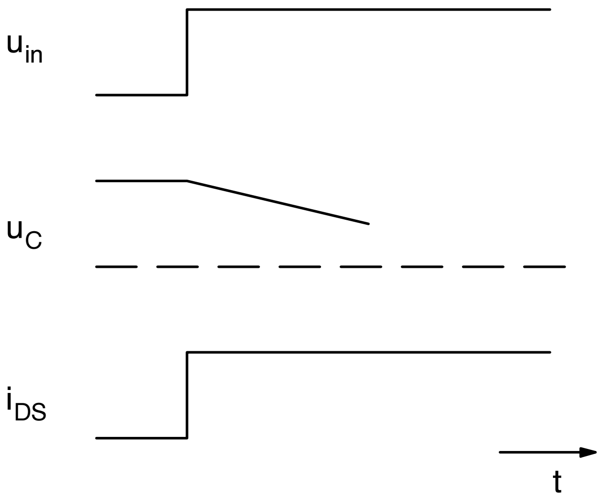
$U_{DD} - u_C \leq (U_{DD} - u_{in} + U_{Tp})/2$

$u_{in} \leq 2u_C - U_{DD} + U_{Tp}$

$$R_{Tn} = \frac{1}{\beta(U_{GSn} - U_{Tn})}$$

$$R_{Tp} = \frac{1}{\beta(-U_{GSp} + U_{Tp})} = \frac{1}{\beta(U_{DD} - u_{in} + U_{Tp})}$$

c. Voltage and current waveforms over time



$i_{DS,Sat} = \beta/2\,(U_{DD} - U_T)^2 = 0{,}32\,\beta\,U_{DD}^2$

Discharging process with constant current

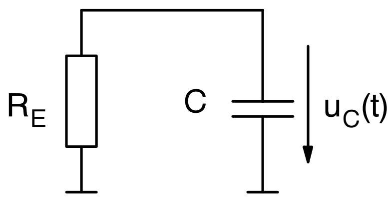
$$i_C(t) = C\frac{du_C}{dt} = -I_0 = const$$

$$u_C(t) = u_C(0) - \frac{1}{C}I_0 \cdot t$$

$$u_C(T) = U_{DD}/2 = U_{DD} - \frac{1}{C}I_0 T$$

$$T = \frac{U_{DD}}{2} \cdot \frac{C}{I_0} = \frac{U_{DD}}{2} \cdot \frac{C}{0,32\,\beta\,U_{DD}^2} = \frac{C}{0,64\,\beta\,U_{DD}}$$

d. Discharge process using an equivalent transistor resistance R_E:



$$u_C(t) = U_{DD}\,e^{-t/R_E C}$$

$$u_C(T) = U_{DD}/2 = U_{DD}\,e^{-T/R_E C}$$

$$T = ln2\,R_E\,C$$

Transistor resistance

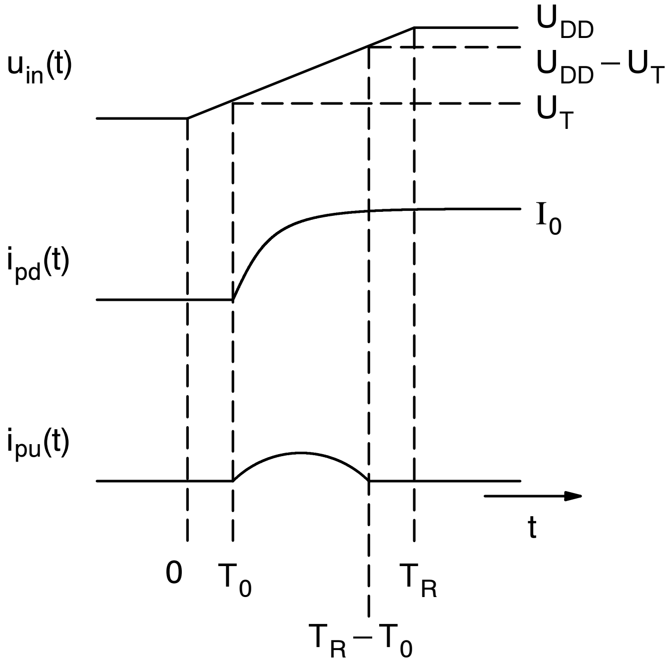
$$R_T\,(U_{GS} = U_{DD}) = \frac{1}{\beta(U_{DD} - U_T)} = \frac{1}{0,8\beta U_{DD}}$$

Using the terms for T from part c. and d. leads to

$$T = ln2\,R_E C = \frac{C}{0,64\beta U_{DD}} = \frac{R_T\,C}{0,8}$$

$$R_E = \frac{1}{0,8\ln 2}\,R_T \approx 1,80\,R_T$$

e. Voltage and current waveforms over time



Simplification: $i_{pu}(t) = 0$

Time unit T_0 :

$$u_{in}(T_0) = U_T = 0,2U_{DD} = U_{DD}\frac{T_0}{T_R}$$

$$T_0 = \frac{U_T}{U_{DD}}\, T_R = 0,2T_R$$

Current: $i_{pd}(t)$ for $T_0 < t < T_R$

$$i_{pd}(t) = \frac{\beta}{2}(u_{in} - U_T)^2 = \frac{\beta}{2}(U_{DD}\frac{t}{T_R} - U_T)^2$$

$$= \frac{\beta}{2}U_{DD}^2\left(\frac{t}{T_R} - \frac{U_T}{U_{DD}}\right)^2$$

$$= a\left(\frac{t}{T_R} - \frac{T_0}{T_R}\right)^2 = a(t' - 0,2)^2$$

$$a = \frac{\beta}{2}U_{DD}{}^2\,;\;\; t' = \frac{t}{T_R} \quad \text{normalized time}$$

Saturation current for $t \geq T_R$:

$$i_{pd}(t) = I_0 = 0,64a$$

Discharging process:

$$i_{pd}(t) = -C\frac{du_C}{dt}$$

$$\int i_{pd}(t)\,dt = -C\int du_C$$

$$\int_{T_0}^{T_R} i_{pd}(t)\,dt + \int_{T_R}^{T} i_{pd}(t)\,dt = -C\int_{U_{DD}}^{U_{DD}/2} du_C$$

Normalization of time $t' = t/T_R$

$$T_R\int_{0,2}^{1} i_{pd}(t')\,dt' + T_R\int_{1}^{T'} i_{pd}(t')\,dt' = CU_{DD}/2$$

$$\int_{0,2}^{1} a(t'-0,2)^2 dt' + \int_{1}^{T'} a\cdot 0,64\,dt' = \frac{CU_{DD}}{2T_R}$$

$$a\frac{(t'-0,2)^3}{3}\Big|_{0,2}^{1} + a\cdot 0,64(T'-1) = \frac{CU_{DD}}{2T_R}$$

$$a\frac{0,8^3}{3} + a\cdot 0,64T' - a\cdot 0,64 = \frac{CU_{DD}}{2T_R}$$

$$T' = 1-\frac{0,8}{3} + \frac{CU_{DD}}{2T_R\cdot 0,64a} = 0,733 + \frac{T_C}{T_R}$$

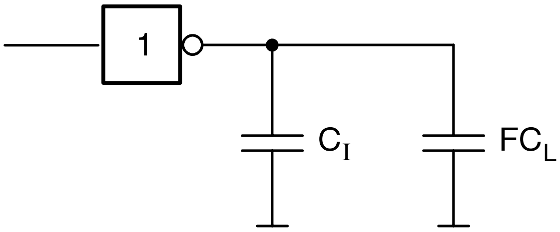
Denormalization

$$T = 0,733\,T_R + T_C$$

Time delay T_C at the input transition according to part c.

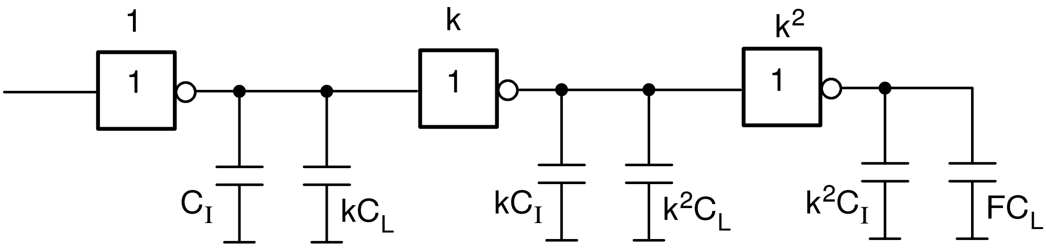
Exercise 2.5

a. N=1



$$T_{D,1} = 2\tau_L + F\tau_L = (2 + F)\tau_L$$

b. Sketched for N=3



$$\begin{aligned} T_{D,N} &= (2\tau_L + k\,\tau_L) \\ &\quad + \frac{1}{k}(2k\,\tau_L + k^2\,\tau_L) \\ &\quad + \frac{1}{k^2}(2k^2\,\tau_L + k^3\,\tau_L) \\ &\quad + \ldots \\ &\quad + \frac{1}{k^{N-1}}(2k^{N-1}\,\tau_L + F\,\tau_L) \\ &= (N-1)(2 + k)\,\tau_L + 2\tau_L + \frac{F}{k^{N-1}}\tau_L \end{aligned}$$

$$\frac{\partial T_{D,N}}{\partial k} = (N-1)\tau_L - (N-1)\frac{F}{k^N}\,\tau_L \overset{!}{=} 0$$

$$k^N = F$$

$$k = F^{\frac{1}{N}}$$

$$T_{D,N,opt} = N\,(2 + F^{\frac{1}{N}})\tau_L$$

$F = 1000$

N	k	$T_{D,N} / \tau_L$	$A_{Si,N}/A_{Si,1}$	$T_{D,N} \cdot A_{Si,N} / \tau_L$
1	1000	1002	1	1002
2	31,6	67	33	2194
3	10,0	36	111	3996
4	5,62	30	216	6577
5	3,98	30	335	10024
6	3,16	31	463	14310
7	2,68	33	595	19480

The product $T_{D,N} \cdot A_{Si,N}$ increases with N, because the increase in the area needed is larger than the decrease of the time delay. For the product, the following expression holds approximately:

$$N \cdot F(1 + \frac{3}{k-1})\, \tau_L$$

Exercise 2.6

a. Let x_1, x_2 be equally distributed:
$P(x_1{=}0) = P(x_1{=}1) = 0.5$; $P(x_2{=}0) = P(x_2{=}1) = 0.5$.
NAND2: $P(y{=}1) = 0.75$

x_1 x_2	y
0 0	1
0 1	1
1 0	1
1 1	0

b.

				NAND2		NOR2	
$x_1(t)$	$x_1(t-1)$	$x_2(t)$	$x_2(t-1)$	y(t)	y(t−1)	y(t)	y(t−1)
0	0	0	0	1	1	1	1
0	0	0	1	1	1	1	0 (*)
0	0	1	0	1	1	0	1 (*)
0	0	1	1	1	1	0	0
0	1	0	0	1	1	1	0 (*)
0	1	0	1	1	0 (*)	1	0 (*)
0	1	1	0	1	1	0	0
0	1	1	1	1	0 (*)	0	0
1	0	0	0	1	1	0	1 (*)
1	0	0	1	1	1	0	0
1	0	1	0	0	1 (*)	0	1 (*)
1	0	1	1	0	1 (*)	0	0
1	1	0	0	1	1	0	0
1	1	0	1	1	0 (*)	0	0
1	1	1	0	0	1 (*)	0	0
1	1	1	1	0	0	0	0

Note: Changes of the output level have been marked by an asterisk.

c. According to the problem the variables x_1 and x_2 should be equally distributed, thus the transition activity function α is:

$$\alpha = \frac{6}{16}$$

d. $P(x_1) = P_1, P(x_2) = P_2$

$$\begin{aligned}\alpha(\gamma) = {} & (1-\gamma)\cdot P_1\cdot(1-\gamma)\cdot P_2 + (1-\gamma)\cdot P_1\cdot\gamma\cdot P_2 + \\ & (1-\gamma)\cdot P_1\cdot\gamma\cdot P_2 + \gamma\cdot P_1\cdot(1-\gamma)\cdot P_2 + \\ & (1-\gamma)\cdot P_1\cdot(1-\gamma)\cdot P_2 + \gamma\cdot P_1\cdot(1-\gamma)\cdot P_2\end{aligned}$$

$$\begin{aligned}\alpha(\gamma) &= 2\cdot((1-\gamma)^2 + 2\cdot(1-\gamma)\gamma)\cdot P_1\cdot P_2 \\ &= 2\cdot(1-\gamma^2)\cdot P_1\cdot P_2\end{aligned}$$

using $P_1 = P_2 = 0.5$ and $\gamma = 0.75$, α can be calculated as

$$\alpha(0.75) = \frac{7}{32}.$$

e. All results are the same (see function table in part b).

f. The transition activity factor depends on the truth table of the logical function, the correlation between variables, and the transition probability for the variables.