Solutions for Chapter 7

Exercise 7.1

a. Critical delay of a PE

Critical delay of DFT-PE as in Figure 7.2.2:

The DFT-PE consists of a complex multiplier and a complex adder for accumulation. Using the structure of Figure 7.2.3 b) for the complex multiplication, the critical path consists of an m_{bit} real addition, an $m_{bit} \times m_{bit}$ real multiplication, a 2 m_{bit} real addition, and a 3 m_{bit} accumulation. The critical delay calculates to

$$T_{DFT-PE,crit} = m_{bit} \cdot T_0 + 3m_{bit} \cdot T_0 + 2m_{bit} \cdot T_0 + 3m_{bit} \cdot T_0 = 9m_{bit} \cdot T_0$$

Critical delay of butterfly PE as in Figure 7.3.2:

The butterfly PE consists of two complex adders and a complex multiplier with one adder and the multiplier in the critical path. Using the structure of Figure 7.2.3 b) for the complex multiplication, the critical path consists of two m_{bit} real additions, an $m_{bit} \times m_{bit}$ real multiplication, and a 2 m_{bit} real addition. The critical delay calculates to

$$T_{FFT-PE,crit} = m_{bit} \cdot T_0 + m_{bit} \cdot T_0 + 3m_{bit} \cdot T_0 + 2m_{bit} \cdot T_0 = 7m_{bit} \cdot T_0$$

b. Throughput using one PE

DFT-PE of Figure 7.2.2: Computation of one sample requires N DFT-PE processing steps.

$$R_{T,DFT-1} = \frac{1}{N} \cdot \frac{1}{T_{DFT-PE,crit}} = \frac{1}{64} \cdot \frac{1}{9 \cdot 8 \cdot 0.5ns} = 0.434 \ Msamples/s$$

FFT-PE of Figure 7.3.2: Computation of two samples requires $log_2 N$ butterfly-PE processing steps.

$$R_{T,FFT-1} = \frac{2}{\log_2 N} \cdot \frac{1}{T_{FFT-PE,crit}} = \frac{2}{6} \cdot \frac{1}{7 \cdot 8 \cdot 0.5ns} = 11.905 \ Msamples/s$$

c. Throughput using 1-D array of PEs

1-D DFT array of Figure 7.2.1: N DFT-PEs compute N samples concurrently.

$$R_{T,DFT-1D} = N \cdot R_{T,DFT-1} = 27.776 \ Msamples/s$$

1-D FFT array of Figure 7.3.5: Horizontal projection of 2-D array; $N/2$ butterfly PEs compute N samples concurrently.

$$R_{T,FFT-1D} = \frac{N}{2} \cdot R_{T,FFT-1} = 380.96 \ Msamples/s$$

1-D FFT array of Figure 7.3.10: Vertical projection of 2-D array; $log_2 N$ butterfly PEs compute 2 samples concurrently.

$$R_{T,FFT-1D} = \log_2 N \cdot R_{T,FFT-1} = 71.429 \ Msamples/s$$

Exercise 7.2

a. Real-valued input sequence to DFT-PE:

The costs of the DFT-PE reduce to 2 multipliers, 2 adders from 4 multipliers, 4 adders (complex multiplier structure 7.2.3 a) or 3 multipliers, 5 adders (complex multiplier structure 7.2.3 b), respectively.

b. Real-valued and even input sequence to DFT-PE:

$$y(k) = \sum_{m=0}^{N-1} w^{mk} x(m); \quad x(i) = x(N-i); \quad Equation\ (7.2.5): \ w^i = (w^{N-i})^*;$$

$$y(k) = x(0) + \sum_{m=1}^{N/2-1} w^{mk} x(m) + w^{(N/2)k} x(N/2) + \sum_{m=N/2+1}^{N-1} w^{mk} x(m)$$

$$= x(0) + \sum_{m=1}^{N/2-1} w^{mk} x(m) + w^{(N/2)k} x(N/2) + \sum_{m=1}^{N/2-1} w^{mk^*} x(m)$$

$$= x(0) + w^{(N/2)k} x(N/2) + \sum_{m=1}^{N/2-1} (w^{mk} + w^{mk^*}) x(m)$$

$$= x(0) + (-1)^k x(N/2) + \sum_{m=1}^{N/2-1} 2\,\mathrm{Re}[w^{mk}]\, x(m)$$

With the assumption of a real-valued and even input sequence, also the coefficients become real-valued. The costs of the DFT-PE reduce to 1 multiplier, 1 adder.

c. Real-valued and even input sequence to FFT-PE:

The costs of the butterfly PE cannot be reduced in a similar way as the irregular computation flow of the fast algorithm does not lead to the elimination of imaginary values.

Exercise 7.3